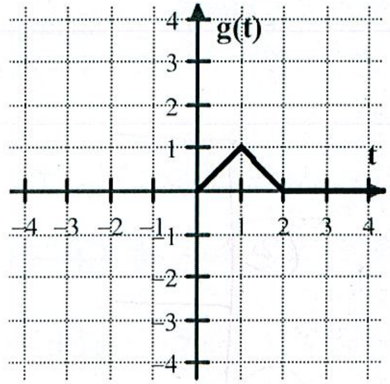
Use Laplace transforms to solve the initial value problem $y'' + 2y' + 2y = g(t)$, $y(0) = 3$, $y'(0) = -4$ **SCORE: _____ / 30 PTS**

where $g(t)$ is the function whose graph is shown on the right.



NOTE: $g(t)$ consists of three pieces, all of which are linear functions.

$$\boxed{g(t) = \begin{cases} t, & 0<t<1 \\ 2-t, & 1<t<2 \\ 0, & t>2 \end{cases}} = t + u(t-1)(2-t-t) + u(t-2)(0-(2-t))$$

2

$$= \boxed{t + u(t-1)(2-2t) + u(t-2)(t-2)} \quad 2$$

$$\mathcal{L}\{g(t)\} = \frac{1}{s^2} + e^{-s}\mathcal{L}\{2-2(t+1)\} + e^{-2s}\mathcal{L}\{(t+2)-2\}$$

$$= \frac{1}{s^2} + e^{-s}\mathcal{L}\{-2t\} + e^{-2s}\mathcal{L}\{t\}$$

$$= \boxed{\frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}} \quad 2$$

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + 2s\mathcal{L}\{y\} - 2y(0) + 2\mathcal{L}\{y\}$$

$$= \underbrace{(s^2+2s+2)\mathcal{L}\{y\}}_{2}\ \underbrace{-3s-2}_{2} = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$$

$$\mathcal{L}\{y\} = \frac{3s+2}{s^2+2s+2} + \frac{1}{s^2(s^2+2s+2)} + \frac{1}{s^2(s^2+2s+2)}(-2e^{-s}) + \frac{1}{s^2(s^2+2s+2)}e^{-2s}$$

2

$$\mathcal{L}^{-1}\left\{\frac{3s+2}{(s+1)^2+1}\right\} = \mathcal{L}^{-1}\left\{\underbrace{\frac{3(s+1)-1}{(s+1)^2+1}}_{2}\right\} = \underbrace{3e^{-t}\cos t - e^{-t}\sin t}_{2}$$

$$\frac{1}{s^2(s^2+2s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C(s+1)+D}{(s+1)^2+1}$$

$$1 = As((s+1)^2+1) + B((s+1)^2+1) + Cs^2(s+1) + Ds^2$$

$s=0$: $\quad 1 = 2B \longrightarrow \underline{B = \frac{1}{2}}$ 2

COEF OF s: $\quad 0 = 2A + 2B \longrightarrow A = -B = \underline{-\frac{1}{2}}$ 2

COEF OF s^3: $\quad 0 = A + C \longrightarrow C = -A = \underline{\frac{1}{2}}$ 2

$s=-1$: $\quad 1 = -A + B + D \longrightarrow D = 1 + A - B = \underline{0}$ 2

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+2s+2)}\right\} = \underbrace{-\frac{1}{2} + \frac{1}{2}t + \frac{1}{2}e^{-t}\cos t}_{2} = \frac{1}{2}(t - 1 + e^{-t}\cos t)$$

$$y = \begin{cases} 3e^{-t}\cos t - e^{-t}\sin t + \frac{1}{2}(t-1+e^{-t}\cos t) & 2 \\ - u(t-1)(t-2+e^{-(t-1)}\cos(t-1)) + \frac{1}{2}u(t-2)(t-3+e^{-(t-2)}\cos(t-2)) & 2 \end{cases}$$